

# DAWSON COLLEGE

## DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION

CALCULUS-III

May 24, 2016

Time: 2:00 pm-5:00 pm

Instructor: A. Panait, T. Kengatharam

---

Name:

ID:

---

**Instructions:**

- Translation and regular dictionaries are permitted.
- Scientific non-programmable calculators are permitted.
- Print your name and ID in the provided space.
- This examination booklet must be returned intact.

**This examination consists of 20 questions. Please ensure that you have a complete examination before starting.**



(3) [5 marks] Approximate the sum of the convergence series <sup>1</sup>

(4) [5 marks] Evaluate the integral  $\int_0^1 xe^{-x^3} dx$  as an infinite series. (Hint: You may use  $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$ )

- (5) [5 marks] Consider the curve with parametric equations  $x = e^t; y = te^t$ .  
Find  $\frac{dy}{dx}$ . For which values of  $t$  is the curve concave upward?

- (6) [5 marks] Sketch the curve with polar equation  $r = 1 - \cos \theta$  for  $0 \leq \theta < 2\pi$ .

(7) [5 marks] Find the equation of the tangent line to the curve with parametric equations  $x = 1 + \sqrt{t}$ ;  $y = e^{t^2}$  at the point  $(2; e)$ .

(8) [5 marks] Find the arc length of the curve  $\underline{r}(t) = (\cos t; \sin t; \ln(\cos t))$  for  $0 \leq t \leq \pi/4$ .

(9) [5 marks] Show that the curvature of a circle with radius  $a$  is  $\frac{1}{a}$ .

(10) [5 marks] Find the equation of the osculating plane to the curve  $\underline{r}(t) = (t; t; t)$  at  $(1; 1; 1)$ .

(11) [5 marks] Study the continuity of

$$f(x; y) = \begin{cases} \frac{xy - y}{x^2 + y^2} & \text{if } (x; y) \neq (0; 0) \\ - & \text{if } (x; y) = (0; 0) \end{cases} .$$





- (13) [5 marks] Find all critical points of  $f(x; y) = 3y^2 - 2y - 3x + 6xy$  and classify them.

- (14) [5 marks] If a particle with mass  $m$  moves with position vector  $\underline{r}(t)$ , then its angular momentum is defined by  $\underline{L}(t) = m\underline{r}(t) \times \underline{v}(t)$  and its torque as  $\underline{\tau}(t) = m\underline{r}(t) \times \underline{a}(t)$ , where  $\underline{v}(t)$  and  $\underline{a}(t)$  are the particle's velocity and acceleration respectively. Show



- (17) [5 marks] Find the volume of the solid that lies inside the sphere  $x^2 + y^2 + z^2 = 16$  and outside the cylinder  $x^2 + y^2 = 4$ .

- (18) [5 marks] Compute the volume of the tetrahedron bounded by the plane  $x + 2y + 3z = 6$  and the three coordinate planes.

- (19) [5 marks] Using cylindrical coordinates evaluate  $\int \int \int_E x \, dv$  where  $E$  is the solid that lies within the cylinder  $x^2 + y^2 = 1$ , above the plane  $z = 0$  and below the cone  $z = 4x + 4y$ .

(20) [5 marks] Prove that  $\int \int \int_E z e^{(x^2+y^2+z^2)^6} dV \leq 0$ , where  $E$  is the lower hemisphere  $\{(x; y; z) \mid x^2 + y^2 + z^2 \leq 1; z \leq 0\}$ .

(Hint: you may use the spherical coordinates  $x = \sin \theta \cos \phi$ ;  $y = \sin \theta \sin \phi$ ;  $z = -\cos \theta$  for which  $dV = \sin \theta \, d\theta \, d\phi \, dr$ .)