Name: .....

Student ID: .....

DAWSON COLLEGE

MATHEMATICS DEPARTMENT

FINAL EXAMINATION

Calculus II Social Science / Commerce 201-203-DW

Instructors: A. Hariton, M. Sohrabi, O. Veres

Date: Thursday, May 19, 2011
Time: 14:00 - 17:00

1. [20 marks] Evaluate the following integrals.

a) $\int x^2\sqrt{x-1}dx$

b) $\int x^2 \ln(2x)dx$

c) $\int \frac{5x^2-2x+9}{(x-1)(x^2+3)}dx$

d) $\int \cos 4x(\sin 4x + 3)^5 dx$

2. [5 marks] Use the limit definition (Riemann Sums) of the definite integral to evaluate

$$\int_0^3 (2x^2 - 3x + 7)dx.$$

3. [5 marks] The weekly marginal cost function associated with producing a unit is given by the function

$$C'(x) = 0.00003x^2 - 0.002x + 10$$

where $C'(x)$ is measured in dollars/unit, and x denotes the number of units produced. The weekly fixed cost incurred is $ 542. Find t

4. [10 marks] For a certain commodity the demand equation is $p = -0.1x^2 - x + 104$ and the supply equation is $p = 0.4x^2 + 80$. The price is set at the equilibrium. Find

   a) Consumers' surplus.

   b) Producers' surplus.

5. [5 marks] Use Simpson's Rule with $n = 4$ to approximate the value of the definite integral.

$$\int_0^4 \sqrt{x^3 + 2}\,dx$$

   Round your answer to four decimal places.

6. [5 marks] Find the area of the region completely enclosed by the graphs of the functions $y = x^2 - x - 6$ and $y = 2x - 2$.

7. [5 marks] Find the limit.

$$\lim_{x \to 0} \frac{4\ln(x + 1) + 5\cos x - 4x - 5}{e^{3x} - 3x - 1}$$

8. [5 marks] Evaluate the integral, if it converges.

$$\int_{-\infty}^{2} 4xe^{(-x^2+6)}\,dx$$

9. [5 marks] Verify that $y = xe^{4x}$ is a solution to the differential equation.

$$y'' - 4y' = 4e^{4x}$$

10. [5 marks] Use separation of variables to find the particular solution of the differential equation

$$y' = \frac{y^2}{(y^4 - 2y^3)(x + 1)^2}$$

   subject to the initial condition $y(1) = 1$.

INFORMATION PAGE

$$\sum_{k=1}^{n} 1 = n$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$CS = \int_0^{\bar{x}} D(x)dx - \overline{px} \qquad PS = \overline{px} - \int_0^{\bar{x}} S(x)dx$$

$$\int_a^b f(x)dx = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

where $\Delta x = \frac{b-a}{n}$

$$\int_a^b f(x)dx = \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+4f(x_{n-1})+f(x_n)]$$

where $\Delta x = \frac{b-a}{n}$ and n is even.

ANSWERS

1. **a)** $\frac{2}{7}(x-1)^{\frac{7}{2}} + \frac{4}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}}) + C$ **b)** $\frac{1}{3}x^3 \ln(2x) - \frac{1}{9}x^3 + C$
   c) $3\ln|x-1| + \ln(x^2+3) + C$ **d)** $\frac{1}{24}(\sin 4x + 3)^6 + C$

2. $\frac{51}{2}$

3. $C(300) = 3722$

4. $\bar{x} = 6$, $\bar{p} = 94.4$, $CS = 32.4$, $PS = 57.6$

5. **14.7772**

6. $\frac{125}{6}$

7. -1

8. $-2e^2$

10. $\frac{1}{3}y^3 - y^2 = -\frac{1}{x+1} - \frac{1}{6}$

11. $4 + 6(x-1) + \frac{3}{2}(x-1)^2 + 2(x-1)^3 - \frac{3}{4}(x-1)^4$

12. $\frac{109}{18}$

13. $-\frac{4}{75}$

14. **a) convergent by the Integral Test** **b) convergent by the Comparison Test**
    c) divergent by the Test for Divergence.