# Dawson College <br> Department of Mathematics <br> Final Examination <br> Winter 2011 

## CalculusI (201-NYA-05)

Date: May 16, 2011, 9:30am

Examiners: J. Requeima, O. Veres

Student Name: $\qquad$

Student I.D.: $\qquad$

- Print your name and student ID number in the space provided above.
- All questions areto be answered directly on the examination paper in the provided space.
- Non-programmabl e cal cul ators are permitted.

This examination consists of 13 questions, 18 pages and 100 points. Please ensure that you have a complete examination before starting. This exam must be returned intact.

Question 1. Evaluate the following limits:
(a) (4 marks)
$\lim _{x \rightarrow-}$
(c) (4 marks)

$$
\lim _{x \rightarrow 3} \frac{x-3}{4-\sqrt{13+x}}
$$

Question 2. (6 marks) Show that the following function is continuous everywhere:

$$
f(x)=\begin{array}{ccc}
x^{2}-3 x+5 & \text { if } & x<2 \\
3 & \text { if } & x=2 \\
\frac{x^{2}+2 x+7}{x^{2}+1} & \text { if } & x>2
\end{array}
$$

Question 3. (6 marks) Find the derivative of $f(x)=5 x-2 x^{2}$ using the limit definition of the derivative.

Question 4. Find the derivatives of thefollowing functions:
(a) (4 marks)

$$
f(x)=\frac{x^{3}}{3}+3 \sqrt{x}-\frac{1}{x}+\sin x
$$

(b) (5 marks)

$$
f(x)=\frac{\sqrt{x^{3}-3 x}}{\cos \left(3 x^{2}-1\right)}
$$

(c) (4 marks)

$$
f(x)=x^{1 / 3} \ln x+\arcsin
$$

Question 5. (6 marks) Find the equation of the tangent lineto $f(x)=e^{x^{2}} \cos x$ at $x=0$.

Question 6. (6 marks) Use logartithmic differentiation to find the derivative of

$$
f(x)=(x-2)^{14}\left(3 x^{2}+5 x\right)^{p} \overline{x^{2}-2}
$$

Question 7. ( 6 marks) Find $\frac{d y}{d x}$ given

$$
(x y)^{3}=3 x^{2} y+y
$$

Question 8. ( 6 marks) The demand equation for $x$ units of a product is $x=200-0.02 p^{2}$ where $p$ is in the price in dollars.
(a) Find the elasticity of demand function given $E(p)=-\frac{p f^{\prime}(p)}{f(p)}$.
(b) Determine if demand is elastic or inelacstic when $\mathrm{p}=\$ 35$.
(c) At what price is elasticity unitary?

Question 9. (6 marks) Suppose the demand equation for a certain product is

$$
300 x^{2}+13 p^{2}=3900
$$

where $x$ represents the number of units in thousands demanded each week when the unit price is $\$ p$. How much is the quantity demanded increasing when the unit price is $\$ 15$ per unit and the unit

Question 10. (5 marks) Find the domain, intercepts, and any asymptotes of

$$
f(x)=\frac{x^{2}+x-2}{x^{2}+2 x-15}
$$

Question 11. Computethefollowing antiderivatives
(a) (3 marks)

Z

$$
\sqrt[3]{x}+3 e^{x}+\frac{1}{x} d x
$$

(b) (3 marks)

$$
\left.\frac{z}{x^{3}+\sqrt{x}-x}\right) d x
$$

Question 12. (8 marks) A rectangular storage contai ner with an open top is to have a volume of $10 \mathrm{~m}^{2}$. The length of the base is twice the width of the base. Material for the base costs $\$ 10$ per square metre. Material for the sides costs $\$ 6$ per square metre. Find the cost of materials for the cheapest such container.

Question 13. Consider the function $f(x)=3 x^{4}-4 x^{3}$
(a) (1 marks) Find the domain of $f$.
(b) (2 marks) Find the $x$ and $y$-intercepts (if any).
(c) (3 marks) Find the intervals where $f$ is increasing and the intervals where $f$ is decreasing.
(d) (1 marks) Find any relative extrema.
(e) (3 marks) Find the intervals where $f$ is concave upward and the intervals where $f$ is concave downward.
(f) (1 marks) Find any inflection points.
(g) (4 marks) Use the above information to sketch the graph of f. Clearly indi cate the coordinates of any points found above.

